Statistical Analyses of weigh-in-motion data for Bridge Live Load Development

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ABSTRACT: This paper discusses the statistical analyses used to derive bridge live loads models for Hong Kong from a 10-year weigh-in-motion (WIM) data. The statistical concepts required and the terminologies adopted in the development of bridge live load models are introduced. This paper includes studies for representative vehicles from the large amount of WIM data in Hong Kong. Different load affecting parameters such as gross vehicle weights, axle weights, axle spacings, average daily number of trucks etc are first analyzed by various stochastic processes in order to obtain the mathematical distributions of these parameters. As a prerequisite to determine accurate bridge design loadings in Hong Kong, this study not only takes advantages of code formulation methods used internationally but also presents a new method for modelling collected WIM data using a statistical approach.

1 INTRODUCTION

The ability to weigh vehicles as they travel at a highway speed has become known as Weigh-in-Motion (WIM) technology. Using a WIM system, virtually a 100% sample of traffic data for statistical purposes can be obtained. The information can be transmitted immediately in real time, or at some future time, to locations remote from the WIM site via conventional communications networks. At present, WIM systems are used for enforcement primarily to identify individual vehicles that are suspected of being in violation of weight or size laws and for locating sites where relatively large numbers of probable weight, speed, or size violations occur. Relevant departments of the Hong Kong SAR Government (known as Hong Kong Government before 1997) decided to install WIM systems in Hong Kong several years ago, and a large amount of WIM data has been obtained. These WIM data can be analyzed to understand the real traffic status for comparison with bridge design live load models in use in Hong Kong to test overloading. These WIM data can also supply basic information, such as traffic flow, and heavy vehicle distribution to plan road networks. Bridge loading models are very closely related to gross vehicle weights, axle weights, axle spacings and average daily number of trucks. If the mathematical distributions of these parameters are obtained accurately, bridge live load models can subsequently then be easily formulated. WIM systems can provide a great amount of real traffic data to assist in determine these parameters. The problem is how to determine these parameters from the measured WIM data. It is impossible to determine these parameters in a load model if the tremendous amount of WIM data is not described statistically.

This paper discusses the methodologies, analytical concepts and the statistical models derived from the analysis of a 10-year WIM database described in (Miao 2001). The statistical analysis approach used in this paper is an effective method to obtain the cumulative distribution function (CDF) of related parameters such as gross vehicle weight, axle weight, axle spacing and so on. These obtained parameters are the base of formulating the bridge live load models (Miao and Chan 2002). This approach can provide a distinct CDF and a required value of any parameter under a given probability level.

2 STATISTICAL PROCEDURE

The statistical procedure adopted to estimate the maximum values of the gross vehicle weight and axle weight in bridge design life includes the following.

2.1 *WIM data collecting and handling*

The system named International Road Dynamics's (IRD) WIM system was installed at seven sites in Hong Kong (Figure 1). Data for the present studies were collected from these WIM sites. For each site, a roadside computer collects signals from the various in-road sensors (e.g. piezo sensors, ground beam

WIM Stations
A: Lung Cheung Road
B: Tolo Highway
C: Tung Mun Highway
D: Kwai Chung Road
E: Island Eastern Corridor
F: Tsing Ma Bridge
G: Lantau Toll Plaza

Figure 1 Hong Kong Weigh-In-Motion (WIM) Locations

systems) and interprets them to create records of the vehicles passing over. The vehicle records can be analysed either on site or in office.

2.2 *Selection of Statistical Processes and Estimation of the corresponding Statistical Parameters*

Many statistical approaches (Benjamin and Cornell 1970) can be used to analyze the collected data. Two steps, commonly used method, can be carried out.

1. Put forth several basic distribution assumptions;
2. Carry out the simulation of these assumptions.

The simulation steps can be summarized as the follows:

1. Put forth statistical assumptions H_i (i = 1,2,…);
2. Choose stochastic variables U_i and carry out statistical analysis and verify which distribution that will fit the random variables;
3. Define a confidence limitation $U_{1\alpha}$, $U_{2\alpha}$ for given random variables. Their probabilities should meet the following requirement:

$$P(U_{1\alpha} < U < U_{2\alpha}) = 1\text{-}\alpha \tag{1}$$

where α is a given probability level.

4. According to the statistical sample, calculate u that belongs to the random variable U;
5. When the values of u are in the area of $U_{1\alpha} < U < U_{2\alpha}$, the statistical assumptions H_i are true

In order to calculate the statistical parameters, expected values and standard deviations, from the given samples of data, the Maximum Likelihood Estimation (MLE) method will be used in this paper.

For a random variable X, with a known probability distribution function $f_X(x)$, and the observed values $x_1, x_2, \ldots, x_n$, in a random sample of size n, the likelihood function of θ, where θ represents the set of unknown parameters, is defined as:

$$L(\theta) = \prod_{i=1}^{n} f_X(x_i \mid \theta) \tag{2}$$

The objective is to maximize $L(\theta)$ for the given data set. This can easily be done by taking m partial derivatives of $L(\theta)$ with respect to where m is the number of parameters, and equating them to zero. The MLE of parameter set θ from the solutions of the equations is then obtained. In this way the greatest probability is given to the observed set of events, provided that the true form of PDF is known.

In order to obtain the mathematical distributions and their related parameters (Rohatgi. 1976) of observed data, researchers have been doing much work. Miao (2001) stated that the following stochastic processes should be considered to analyze the related problem of highway live loads.

i. Normal Distribution (ND) (Norman 1994)
ii. Weibull Distribution (WD) (Norman 1994)
iii. Filtered Weibull Process (FWP) (Norman 1994)
iv. Gamma Distribution (GD) (Norman 1994)
v. Logarithmic Normal Distribution (LND) (Norman 1994)
vi. Extreme Value Distribution –Type I (EVD) (Norman 1994)

vii. Inverse Gaussian Distribution (IGD) (Tweedie 1957 and Seshadri 1993)
viii. Filtered Poisson Process (FPP) (Norman 1994)

2.3 *Grouping of the recorded WIM data*

Recorded WIM data from the IRD system contain a mixture of gross vehicle weight, axle weight, axle spacing and vehicle speeds. This mixture must be divided into several particular separate domains (gross vehicle weight, axle weight, axle spacing, headway, vehicle speed, time interval etc) in accordance with required objectives for statistical analysis. Hence, the present study uses the sub-samples mentioned above for statistical analysis. They can be generally expressed as:

$$X = \{X_i\} \tag{3}$$

where X is the vector of a sample in a measuring time section; $\{X_i\}$ is the ith sample in the time section and i is the number of the cell in objects belong to the same time section and collection position.

The objects mentioned above will be analyzed statistically to obtain the mathematical distribution models of axle weight, gross vehicle weight, axle spacing and time interval. These models form a very important basis in the modeling of bridge design load models. They can provide basic information for concentrated loads, uniformly distributed loading, gross vehicle weight and axle spacing to build a bridge live loading model. Two kinds of traffic status (Miao 2001) are used. The inattentive traffic status will be used for truck loading, and the dense traffic status will be used for lane loading. With a tremendous database of several years and many divergent analytical requirements, there is a need to divide the data into appropriate forms for analysis. This, and the desire to develop a report for bridges from the WIM data from each site, has resulted in the development of statistical parameter indices of a bridge live load models. These parameter indices which will be discussed later are very useful for developing bridge live load models. Stochastic processes (Miao 2001) are used to simulate the WIM data. The commonly used maximum likelihood estimation (MLE) approach will be used here to determine these parameters.

The WIM general data sample must be divided into several special needed domains as mentioned above. These are vehicle gross weight, axle weight, headway, vehicle speed, axle spacing and time interval respectively, because bridge live load models are closely related to these parameters.

2.4 *Simulation of the selected Stochastic Processes*

The selection of statistical models (Hahn and Samuel 1994) is the first step towards simulation of the stochastic processes. The checking method adopted in the present study is the Kolmogorov-Smirnov (K-S) (Hahn and Samuel 1994) approach. It can offer a critical value followed by a given reliable parameter. The advantage of this approach is that all deviations can be obtained between every observed distribution point and theoretical distribution point.

Some commercial statistical packages can be used to check data, e.g. SPSS, SAS and SPSSX (Illinois (1999). These packages can be used to achieve some statistical objectives when studying objects which are normal or for simple problems. The stochastic processes of maximum value of gross vehicle weight or axle weight in a general period is a compound stochastic process based on sub maximum values of every time sub-section. It is troublesome to use such software to check compound stochastic processes. Thus a Fortran program is developed to analyze the WIM data. It can give out expected values, mean values and the K-S under a given reliable parameter.

The Monte Carlo simulation method (Kottegoda 1998) is one of the most commonly used approaches to simulate complex random variables and complex stochastic processes. The advantages of this approach are well suitable for any problem with simple simulating procedure. The procedure is usually repeated to generate a different set of values of the variables in accordance with a specified probability distribution. In this way, a series of solutions is obtained corresponding to various sets of the random variables. The Monte Carlo method is therefore used to simulate the distributions of maximum value stochastic processes of a series of given stochastic processes.

The simulating approach of the filtered Weibull Process can be expressed as follows:

If an observation will be ended when the n^{th} vehicle loading occurs, the time intervals of adjacent loadings can be expressed as $X_1, X_2, \ldots\ldots X_n$. The appearing moment of ith loading is:

$$T_i = \sum_{j}^{i} X_j \text{ , } j=1,2,\ldots,n \tag{4}$$

where, T_1, $T_2, \ldots T_n$ are the appearing moments of loadings in the Weibull Process that their density function can be expressed as $\lambda(t)=\lambda\beta t^{\beta-1}$ and λ , β are the statistical parameters.

The simulating statistical value will be calculated by the following formula:

$$F = \frac{[(n-1)-r][(n-1)\ln T_n - \sum_{i=n-r}^{n-1} \ln T_i - (n-1-r)\ln T_{n-r}]}{r[(n-1-r)\ln T_{n-r} \sum_{i=1}^{n-1-r} \ln T_i]} \quad (5)$$

It is an F distribution and its statistical parameters (expected value and standard deviation) are [2r, 2(n-1-r)], where, r = [(n-1)/2]
The values of MLE of statistical parameters expected value and standard deviation $\hat{\alpha}$ and $\hat{\beta}$ can be calculated by the following formulae.

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n} \ln(T_n / T_i)} \quad (6)$$

$$\hat{\lambda} = \frac{n}{T_n \beta} \quad (7)$$

When the acting time is very short, the complicated problems of the simulations of random processes can be carried out by a Filtered Poisson Process. As the loading time of a vehicle on a bridge is very short, the gross weight stochastic processes on a bridge can be described by a filtered poisson process. The gross weight stochastic process $\{S(t), t\in [0,T]$ can also be expressed by a filtered poisson process.

$$S(t) = \sum_{n=\mathbf{0}}^{N(t)} \omega(t; \tau_n, s_n) \quad (8)$$

where
a) $\{N(t), t\in[0,T]\}$ is a poisson process of parameter λ.
b) Responding function

$$\omega(t;\ \tau_n, s_n) = \begin{cases} s_n, t \in \tau_n \\ \mathbf{0}, t \notin \tau_n \end{cases} \quad (9)$$

where τ_n is the loading time of the n^{th} vehicle, τ_o=0

c) S_n(n=1,2,……) are variables following F(x) which are independent of each other and S_o =0.

The maximum value probability distribution of filtered poisson process can be expressed by the following formula:

$$F_M = exp\{-\lambda T[1-F(X)]\} \quad (10)$$

In which F(X) is the stochastic process mentioned above; λ is the parameter of poisson process, and it can be calculated by the MLE approach; T is the period of requirement.

3 STATISTICAL PROCESS SELECTION

The general specimens of every divided WIM domain assembled from 1986 to1995 are very large. It is impossible to simulate such a large specimen at any one time. Therefore, sub-samples are assembled. These subsamples can be assembled randomly again and again. It is found that the checked samples follow several stochastic processes at the same time if they are not large enough. Thus, the samples have to be enlarged and re-simulated until only one stochastic process passes the checking instantly.

3.1 *Gross Weight*

The gross vehicle weight samples are randomly drawn from the IRD recorded data. There are three kinds of gross vehicle weight samples.

- Drawn randomly from several continuous days' data at an observed site.
- Drawn randomly from several continuous days' data at a site for a small sample first, then another sample from a different continuous days' data at the same site drawn, to make up a bigger sample from these sub-samples.
- Drawn randomly from different sites' several continuous days' data to assemble a sample.

The above samples shall not be assembled from different year's data or the growing rate cannot be calculated. The assembly can be expressed as:

$$G = \{G_i\} \quad (11)$$

where G is the gross weight vector of a sample in a measuring time section; $\{G_i\}$ is the i^{th} sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

To simulate the distribution of gross vehicle weight, a total number of 317,367 vehicles are studied. It is found that the distribution of gross vehicle weight obeys the Inverse Gaussian Distribution (IGD). Its statistical parameters are: Expected Value = 20.04 and Standard deviation = 28.30.

3.2 *Axle Wight*

The principle of assembling the samples of axle weight is the same as for that of the gross vehicle weight. Axle weights that belong to those vehicles whose gross weights are less than 9.0 tons will not be assembled into the checked samples.

$$P = \{P_i\} \quad (12)$$

where P is the axle weight vector of a sample in a measuring time section; $\{P_i\}$ is the i^{th} sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

To simulate the distribution of gross vehicle weight, a total number of 696,181 vehicles are studied. It is found that the distribution of axle weight obeys the Inverse Gaussian Distribution (IGD). Its statistical parameters are: Expected Value = 4.25 and Standard deviation = 7.05.

3.3 *Axle Spacing*

The domains for axle spacing are assembled by those vehicles with their gross weights greater than 9.0 tons and are specified as follows:

$$S = \{S_i\} \tag{13}$$

where S is the axle spacing vector of a sample in a measuring time section; $\{S_i\}$ is the i^{th} sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

A general sample of 125,954 elements assembled from 5 sites is simulated. The results show that the distribution of axle spacing obeys the Lognormal Distribution (LND). Its statistical parameters are: Expected Value = 0.41 and Standard deviation = 1.04.

3.4 *Headway*

The samples for headway drawn randomly from WIM data are simulated. They can be expressed as:

$$H = \{H_i\} \tag{14}$$

where H is the headway vector of a sample in a measuring time section; $\{H_i\}$ is the i^{th} sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position. There are 145,256 vehicles in the samples. The results show that the distribution of the headway obeys the Lognormal Distribution (LND). Its statistical parameters are: Expected Value = 4.9747 and Standard deviation = 1.13675.

3.5 *Time Interval*

The time interval samples are divided into two types. One is inattentive status and the other is dense status. According to the TDHK (1997), the vehicles whose time intervals are less than 2 seconds belong to the dense status whilst the remainder belong to inattentive status. They are generally expressed as:

$$T = \{T_i\} \tag{15}$$

where T is the time interval vector of a sample in a measuring time section; $\{T_i\}$ is the i^{th} sample in the time section and i is the number of the cell in objects of the member of the same time section and collection position.

a) Inattentive Status
   This kind of status will be used for lane loading in the future. It is unnecessary to simulate it, because the IRD system uses the second for the smallest unit to record the time intervals.
b) Dense Status
   In the samples for time intervals of dense status, there are 8,386 elements drawn. The results show that the time interval of Hong Kong WIM data obeys the Gamma Distribution (GD). Its statistical parameters are: Expected value = 1.01 and Standard deviation = 0.12.

4 MAXIMUM VALUE ESTIMATION

After identifying the stochastic processes for each bridge loading related parameters, the distribution parameters of every domain can be obtained. The maximum value of vehicle gross weight and axle weight for a design return period of bridge life can then be determined. For predicting the maximum value of vehicle gross weight and axle weight during the bridge service period, the probability distribution functions of the maximum value stochastic processes of vehicle gross weight stochastic processes and the maximum value of axle weight stochastic processes should be obtained. These processes can provide some information on the gross weight limitation of and the axle weight in future formulated bridge design codes. These distribution functions are considered under inattentive traffic status and dense traffic status respectively. The inattentive traffic status will be used for formulating the codes for standard trucks and the dense traffic status for lane loadings.

4.1 *Loose Status*

a) Gross Weight

It has been mentioned earlier that the distribution of gross weight is known to obey the Inverse Gaussian Distribution andthe time interval distributions of vehicles obey the Gamma Distribution. As the expected value α for the time interval distribution is 1.01, which is very close to 1.0. To simplify the calculation, the time intervals can be approximately described as an exponential distribution with α equal to 1.0. The subsequent density function can then be expressed as

$$f(x) = \lambda exp(-\lambda x) \qquad x > 0 \tag{16}$$

where λ is the distribution standard deviation and its estimated value is 0.12

As the loading time of a vehicle on a bridge is very short, the gross weight stochastic processes on a bridge can be described by a filtered poisson process (Miao 2001).

Taking a week as the observation unit to analyse the maximum distribution of vehicle gross weight, the calculated time area of the maximum distribution of vehicle gross weight should be calculated as follows:

$$T = 7 \times 24 \times 3600 = 604800 \text{ second} \tag{17}$$

Since the parameter λ (standard deviation) = 0.044 (calculated by MLE), the distribution function can be expressed as:

$$F(X) = \Phi[(lnX\text{-}\mu)/\alpha]=\Phi[(lnX\text{-}20.04)/28.03] \qquad (18)$$

The design life of highway bridges in Hong Kong is 120 years. The weekly observation data can be drawn randomly as a sample, and the maximum value of this weekly data can be taken to describe approximately the maximum value of a year. Then the maximum value of design life can be obtained by the following formula (Miao 2001):

$$F_T(X) = exp\{-\lambda T[1\text{-}F_M(X)]\} \qquad (19)$$

where λ=0.044; T=120 years; $F_M(X)$ is a maximum distribution of a year.

It can be seen that Equation 19 is very difficult to solve. The calculation method used here is based on the Monte-Carlo approach. According to the theory of probability, if a function $F(X)$ is the distribution function of a stochastic variable x, $F(X)$ obeys an uniform distribution on area (0,1). According to this principle, many counterfeit functions can be produced by means of the Monte-Carlo approach. These counterfeit random functions will be checked by the Kolmogorov-Smirnov approach.

The procedure is described as follows:

i) Generate 100 stochastic processes on area (0,1) by the Monte-Carlo approach;
ii) Line them from lowest to highest as $W(1)$, $W(2)$,…,$W(100)$;
iii)Obtain F_T(X) according to Equation 19,

$$exp\{-\lambda T[1\text{-}F_M(X)]\}=W(i),\ i=1,2,\ldots,100 \qquad (20)$$

where λ=0.044, T=120years.

iv)Obtain the random counterfeit functions $B(i)$, i =1,2,…,100 from the following,

According to Equation 20,

$$F_M(X_i) = [1+ln\ W(i)]/\ \lambda T, \qquad i=1,2,\ldots,100 \qquad (21)$$

Letting,

$$F_M(X_i)=exp\{-\lambda T[1\text{-}F(Xi)]\}=\mu(i),\ i=1,2,\ldots,100 \qquad (22)$$

If λ=0.044, T=120*years,* then the result:

$$F(X_i)=\{1+ln[\mu(X_i)]\}/\lambda T=\Phi[(lnX_i\text{-}\mu)/\sigma],$$
$$i=1,2,\ldots,100 \qquad (23)$$

Making

$$F(X_i)=Z(i) \qquad i=1,2,\ldots,100 \qquad (24)$$

Functions $Z(1),Z(2),\ldots,Z(100)$ can be obtained respectively using Equation 20 and Equation 24. $A(i)$ can then be obtained through checking the Gaussian distribution form:

$$A(i) = \varphi^{-1}[Z(i)],\ i=1,2,\ldots,100 \qquad (25)$$

With $(lnX\text{-}\mu)/\sigma=A(i)$, then

$$X_i=exp[\mu+\sigma A(i)]=B(i)\ ,\ i=1,2,\ldots,100 \qquad (26)$$

where μ =1.67 *and* σ =0.82.

Now the 100 random counterfeit functions $B(i)$, i = 1,2,…,100, have been obtained. The Kolmogorov-Smirnov approach is used again to check whether Normal, Lognormal, Weibull, Gamma, Extreme Value Type-I and the Inverse Gauss Distribution describe the maximum value distribution of the gross weight. According to the simulated results, the Extreme-Value Type-I distribution describes the data best. The results show that the maximum value distribution of the gross weight stochastic process follows the Extreme-Value Type-I Distribution. That is:

$$F_T(X)=exp\{-exp[-(X\text{-}\alpha)/\beta]\} \qquad (27)$$

where α and β are distribution parameters, that can be obtained by MLE as: α = 137.682, β = 41.784.

b) Axle Weight

As described earlier, the random process of axle weight can be described by the Inverse Gaussian Distribution (IGD). Its estimated statistical parameters are μ = 4.25, α = 7.05.

Axle weight random processes on a bridge can be described as a filtered Poisson process and the maximum value stochastic process distribution is found to be described as an Extreme Value Type-I process using the same approach mentioned above (Equation 27) and the values of MLE are found to be:
α = 75.003, β = 19.359.

4.2 *Dense Traffic Status*

a) Gross Weight

According to WIM data and statistical analysis, when the loading times of vehicles on a bridge are very short, the maximum value stochastic process of vehicle gross weight stochastic processes can be described as a Filtered Weibull Distribution (Miao 2001). The distribution of gross weight cross-section data obeys the Inverse Gaussian Distribution and its statistical parameters are: μ =20.04, σ =*28.30.*

The maximum value distribution in design life of a bridge can also be determined by the Monte-Carlo approach. The steps are the same as that for the inattentive traffic status. The formula is:

$$F_M(X)=exp\{-\lambda T^{\beta}[1\text{-}F(X)]\} \qquad (28)$$

where λ and β are the parameters, T is the design life of bridges.

According to the results of simulation, it can be seen that the extreme value distribution of gross weight under dense traffic status is described as a Weibull Distribution. Its distribution function is:

$$F_T(X)=1\text{-}exp[\text{-}X/\alpha^{\beta}], \qquad X>0 \qquad (29)$$

where α and β are values of the maximum likelihood estimation: α = 358.275, β = 22.229.

b) Axle Weight
The statistical parameters of the distribution of axle weight can be obtained by the same method as that for the gross weight. The maximum value distribution of axle weight under dense traffic status is also described as a Weibull distribution. Its statistical parameters are: α = 225.356, β = 12.340.

4.3 *Maximum Value of Gross Vehicle Weight and Axle Weight*

a) Gross Weight
To substitute the distribution parameters calculated above for Extreme Value Distribution – Type I (Normal 1994):

$$F_T(X) = \exp\{-\exp[-\alpha(X-\beta)]\} \tag{30}$$

the distribution function of gross weight under inattentive traffic status can be expressed as:

$$F_T(X) = exp\{-exp[-137.682(X- 41.784)]\} \tag{31}$$

The distribution function for Weibull Distribution (Norman 1994) is:

$$F(x) = 1\text{-}exp[-(x/\beta)^{\alpha}] \tag{32}$$

Hence, the distribution function of gross weight under dense traffic status can similarly be obtained by substituting the corresponding distribution parameters into Equation 32 as:

$$F_T(X) = 1\text{-}exp[-(X/24.692)^{358.274}] \tag{33}$$

According to these two formulae, the maximum gross weight value in bridge design life under a given probability constant, say 0.95, can be obtained,

By solving Equation 28, the maximum value of gross weight under inattentive traffic status is W_G=41.80t, and by solving Equation 30, the maximum value of gross weight under dense traffic status is W_G=24.70t.

According to the above statistical results, it can be seen that heavy vehicles rarely pass the central areas of cities. The maximum loading of a single vehicle in these areas is far less than that of those in other areas. However, it can be seen that the traffic can be described as dense status. Distributed loading is the best form to describe this kind of traffic.

b) Axle Weight
The axle weight maximum value distribution during bridge design life can be computed as follows:
Under the inattentive traffic status, substitute the corresponding distribution parameters from into Equation 27:

$$F_T(X) = exp\{-exp[-75.003(X\text{-}19.359)]\} \tag{31}$$

Under the dense traffic status, substitute the corresponding distribution parameters into Equation 29:

$$F_T(X) = 1\text{-}exp[-(X/12.34)^{225.356}] \tag{32}$$

By solving Equation 31, the maximum value of axle weight during bridge design life is W_L=19.36t, and by solving Equation 33, the maximum value of gross weight under dense traffic status is W_L=12.40t.

The results of maximum axle weight show that there are not many vehicles with axle-groups found in dense traffic status.

It is interesting to note that the maximum value of gross weight and axle weight during bridge design life, say 120 years, calculated according to the above approach are very close to their legal limitations of Hong Kong which are 42 tons for gross weight and 10 tons for axle weight TDHK (1997).

5 CONCLUSIONS

To meet the demands, the loading carried by a heavy vehicle will be getting heavier and heavier, a bridge design loading should meet the needs of this development. The maximum gross vehicle weight within a bridge design life, say 120 years, should be obtained and considered in the development of bridge design loading models. The statistical analysis approach used in this paper is an effective method to obtain the cumulative distribution function (CDF) of related parameters such as gross vehicle weight, axle weight, axle spacing and so on. These obtained parameters are the base of formulating the bridge live load models. This approach can provide a distinct CDF and a required value of any parameter under a given probability level.

The WIM data from Hong Kong has been statistically analyzed in this paper as an example of the proposed method. The mathematical distributions of gross vehicle weights, axle weights and headway are obtained accordingly. The maximum values of axle weight and gross vehicle weight are compound stochastic processes. Their mathematical distributions are also obtained. Then the maximum gross weights and axle weights within bridge design life under a probability level are then calculated. It is interesting to notice that the maximum value of gross weight and axle weight during bridge design life, say 120 years, calculated according to the above approach are very close to their legal limitations of Hong Kong.

6 ACKNOWLEDGMENT

The work described in this paper was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, PRC (Project No. PolyU 5033/02E). The statistical research and the random process for the gross vehicle, axle weight, axle spacing and so on are based on the

WIM data provided by the Highways Department of the Hong Kong SAR Government. The support provided by the Queensland University of Technology is also gratefully appreciated.

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